Set Relationships

We want to look at how two sets may be related. The relationship between two sets is determined by comparing the elements of the two sets.

Set Equality: We have already used such a relationship when we stated that two sets, perhaps named A and G are equal. To say that two sets are equal means that the two sets have exactly the same members. As an example we will look at $A=\{5,3,7,9,1\}$ and $G=\{x|x \text{ is an odd whole number between 0 and 10}\}$. These sets have exactly the same members. If we know that $y\in A$ then $A=G$ means that $y\in G$, and if we know that $y\in G$ then $A=G$ means that $y\in A$. [Note that A was shown as a set listing where the elements were not given in order. There is no importance to the order of elements in a set. Things are either elements of a set or they are not.]

Here is another set, $C=\{3,6,9,12,15\}$. The relationship between A and C is that they are not equal, and we write $A\neq C$. We can say that because A and C do not have exactly the same members. After all, $6\in C$ but $6\notin A$, or, for that matter, $1\in A$ but $1\notin C$.

Subsets: Here is yet another set, $B=\{9,3\}$. We see that every element of B is also an element of A. In such a case we say that B is a subset of A and we write $B\subseteq A$. More formally, if we are told that we have two sets J and K, and that $J\subseteq K$, then we know that for any $x\in J$ we are certain that $x\in K$. Going back to look at our set C, we see that B⊆C. But we also have that A is not a subset of C because 5∈A and yet 5∉C. Just that one case is enough to say A is not a subset of C, which we write as A⊈C.

Remember that for the sets we have seen, A=G. Is A a subset of G? By the definition of subset the answer must be yes. The elements of the two sets satisfy the requirement that every element of A is also an element of G. Therefore, A⊆G. To say A=G is a <u>stronger</u> statement than to say that A⊆G because equality tells us more about the relationship than does subset. We might notice, however, that we could also say that G⊆A because every element of G is also an element of A. Putting the two relationships together, that is, to say A⊆G <u>and</u> G⊆A, is the same as saying A=G.

Proper Subsets: Returning to the statement B⊆A, we note that A⊈B. Our example element might be 7 because 7∈A but 7∉B. We would like to say something a bit stronger about the relationship between A and B than just to say B⊆A. That slightly stronger statement is that B is a proper subset of A, which we write as B⊂A. More formally, B⊂A means that every element of B is also an element of A <u>and</u> there must be at least one element of A that is not in B. Naturally we have the symbol ⊄ to mean "is not a proper subset of".

Supersets: Though rarely used, mathematic terminology allows us to read some statements backwards, and then, it introduces some new symbols so that we can write things this way. For example, we read the symbols B⊆A, from left to right, as “B is a subset of A”. However, we can read it right to left as “A is a superset of B”. Then, if we want to write “A is a superset of B” but do so left to right, we introduce a new symbol, namely, ⊇,to indicate “is a superset of”. With that symbol we write A⊇B. Of course, just to complete the symbols we also have ⊉ for “is not a superset of”, ⊃ for “is a proper superset of”, and ⊅ for “is not a proper superset of”.

Disjoint sets: So far, the sets we have been using have shared at least one element. What if we have the set D={2,6,10,14}. We see that D and A do not share even a single element. In that case we say that D and A are disjoint sets. We can also see that B and D are disjoint sets. What about C and D? C≠D, C⊈D, D⊈C, and C and D are not disjoint, this latter being the result that they share at least one element, in this case 6∈C and 6∈D. To say that two sets are disjoint is to tell us just a little bit more than to say that the two sets are not equal.